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# Monte Carlo Study of Binary Copolymerizations. 2. The Determination of Reactivity Ratios, Applications 

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## ABSTRACT

This paper discusses an improved and efficient version of the MEM $\varnothing$ RY- 7 computer program described in a previous paper. The program, called MEMøRY-7/1, implements a Monte Carlo algorithm to compute reactivity ratios in binary irreversible copolymerizations. For the copolymerization of acrylic acid and methyl acrylate, the computed $r_{1}, r_{2}$ values ( $r_{1}=1.374$, $r_{2}=1.038$ ) compare quite well with experimentally determined ones ( $r_{1}=1.4, r_{2}=1.0$; Eldridge and Treloar).

## INTRODUCTION

In the previous paper dedicated to this problem [1], we proposed interpreting the sequence distribution as a chemical dictionary:


The chemical features of the monomers are properly taken into account through grammar. We feel that the diagram (1) is commutative,
i.e., one can reverse the direction of the arrows. In this case the chemical dictionary can be used to determine the reactivity ratios which are memorized by the macromolecule and are "written" in the corresponding sentence.

It seems to be right to interpret the blocks $\left.+\mathrm{M}_{1}\right)_{1},\left(-\mathrm{M}_{2}\right)_{1}$ as punctuations and the blocks $\left.+M_{1}\right)_{n \geq 2},\left(-M_{2}\right)_{n \geq 2}$ as words because our Monte Carlo simulations [2, 3] of copolymerization pointed out that the monads are very sensitive to the perturbations of $r_{1}, r_{2}$ values, while longer blocks are stable. Thus the meaning of the sentence lies in the blocks with $\mathrm{n} \geq 2$, and the variability of the blocks with $n=1$ can be regarded as spelling mistakes.

We note that similar ideas are used in molecular biology $[4,5]$.
In order to obtain the reactivity ratios for a pair of monomers which copolymerize following the irreversible, ultimate effect kinetics, we have developed the following strategy: one computes all $r_{1}, r_{2}$ pairs which generate the macromolecules with the same composition $\mathrm{F}_{1}$ or $\mathrm{F}_{2}$ for a given feed composition $\mathrm{f}_{1}$ or $\mathrm{f}_{2}$. One selects the appropriate $r_{1}, r_{2}$ values using one of the following two methods:

1. One takes into account the information concerning the sequence distribution, knowing that the macromolecules discriminate quite well in the sequence distribution space [6].
2. One considers the composition of the copolymers in a manner detailed below.

Monte Carlo algorithms were previously used to study the reactivity in polymer modification reactions (see Ref. 7 and the references cited therein).

## THE MEM $\emptyset$ RY-7/1 PROGRAM

The previous version of the program [1] took about 2 hr to study a macromolecule. The present version of the program significantly improves the computing efficiency: $20 \mathrm{~min} /$ macromolecule.

The input data are as follows: N (polymerization degree), NA (the number of $\mathrm{M}_{1}$-type mers), NB (the number of $\mathrm{M}_{2}$-type mers in the macromolecule), and $C=f_{2} / f_{1}$.

The computing scheme uses the Monte Carlo model of binary irreversible copolymerization [8].

Briefly, the algorithm implemented by MEM $\varnothing$ RY-7/1 is:

1. An $\alpha$ and $\beta$ pair is considered $\left(\alpha=r_{1} /\left(r_{1}+C\right), \beta=r_{2} /\left(r_{2}+C^{-1}\right)\right)$.
2. With the considered $\alpha$ and $\beta$ values, a macromolecule is simulated and characterized by composition ( $\mathrm{A}=$ number of $\mathrm{M}_{1}$ mers, $\mathrm{B}=$ number of $\mathrm{M}_{2}$ mers) and sequence distribution.
3. If $A \pm 5=N A$ and $B \pm 5=N B$, information concerning the generated macromolecule is stored; otherwise it is not.
4. The $\beta$ value is advanced by an amount $10 / \mathrm{N}$ while $\alpha$ is kept fixed; the algorithm is continued with Point 2. When $\beta=1$, the algorithm is continued with Point 5.
5. The $\alpha$ value is advanced by an amount $10 / \mathrm{N}, \beta$ is set $\beta=1 / \mathrm{N}$, and the algorithm is continued with Point 2. When $\alpha=1$, the computing procedure is stopped.

The flow chart of MEM $\varnothing$ RY $7 / 1$ is shown in Fig. 1.

## Observation

The program computes $\alpha$ and $\beta$ with an accuracy of $20 / \mathrm{N}$. Because $\alpha$ and $\beta$ are handled with three decimal digits and random numbers with eight decimal digits, the MEM $\varnothing$ RY $7 / 1$ results differ slightly when compared with the same results obtained by means of the MEMФRY 3 program.

## APPLICATIONS: ACRYLIC ACID/METHYL ACRYLATE COPOLYMERS

In order to illustrate the method described, the copolymerization of acrylic acid and methyl acrylate is studied. The necessary experimental data are taken from Ref. 9 and collected in Table 1.

The representative $r_{1}, r_{2}$ values obtained by means of MEM $\emptyset R Y$ $7 / 1$ are collected in Table 2.

In order to select the appropriate reactivity ratios for the acrylic acid/methyl acrylate pair, the following strategy is used.

For each pair of compositions $(f, F)_{I}$, the set $S_{I}$ is computed:

$$
\begin{array}{ll}
S_{I}=\left\{\left(r_{1}, r_{2}\right)_{J, I}\right\} & I=1,2, \ldots, N \\
J=1,2, \ldots, N_{I}
\end{array}
$$

I indexes the different feed compositions used to synthesize the copolymer considered (the other conditions are fixed), and J indexes the reactivity ratio pairs which gave the same copolymer composition.

Due to an error of experimental composition and to the simplification of the mathematical model, we must consider as definitions that

$$
\left(\mathrm{r}_{1}, \mathrm{r}_{2}\right)_{\mathrm{I}}=\left(\mathrm{r}_{1}, \mathrm{r}_{2}{ }^{\prime}\right)_{\mathrm{I}^{\prime}}
$$

if and only if


FIG. 1. Flow chart of MEMøRY - 7/1 program.

TABLE 1. The Input Data ( $\mathrm{N}=1000$ )

| No. | C | NA | NB |
| :--- | :--- | :--- | :--- |
| 1 | $19.0 / 81.0$ | 857 | 143 |
| 2 | $27.6 / 72.4$ | 772 | 228 |
| 3 | $28.4 / 71.6$ | 756 | 244 |
| 4 | $41.1 / 58.9$ | 615 | 385 |
| 5 | $59.0 / 51.0$ | 561 | 439 |
| 6 | $54.2 / 45.8$ | 491 | 509 |
| 7 | $66.0 / 34.0$ | 411 | 589 |
| 8 | $69.5 / 30.5$ | 326 | 674 |

$$
\begin{equation*}
r_{1} \pm \epsilon_{1}=r_{1}^{\prime} \quad \text { and } \quad r_{2} \pm \epsilon_{2}=r_{2}^{\prime}, \quad 0 \leq \epsilon_{1}, \epsilon_{2} \leq a \tag{2}
\end{equation*}
$$

$\left(r_{1}, r_{2}\right)_{I}$ were computed using the $(f, F)_{I}$ pair, and $\left(r_{1}{ }^{\prime}, r_{2}{ }^{\prime}\right)^{\prime}$, using the pair $(f, F)_{I^{\prime}}$.

The value of $a$ is of the order of magnitude of 3 s , s being the standard error of the experimental copolymer composition.

Using Definition (2), we perform the intersections (3):

$$
\begin{equation*}
\bigcap_{\mathrm{I}=1}^{\mathrm{N}} \mathrm{~S}_{\mathrm{I}}=\left\{\left(\mathrm{r}_{1}, \mathrm{r}_{2}\right)_{K}\right\}, \quad \mathrm{K}=1, \ldots, \mathrm{~L} \tag{3}
\end{equation*}
$$

K indexes the reactivity ratio pairs which obey Relation (2).
The desired $r_{1}, r_{2}$ values are obtained as an average of the $r_{1}$, respectively $r_{2}$, values common (according to Relation 2) to the $N$ sets of computed reactivity ratios which gave the same copolymer compositions. Thus

$$
\begin{equation*}
\mathbf{r}_{1}=\frac{1}{\mathrm{~L}} \sum_{\mathrm{K}} \mathrm{r}_{1} \quad \text { and } \quad \mathrm{r}_{2}=\frac{1}{\mathrm{~L}} \sum_{\mathrm{K}} \mathrm{r}_{2} \tag{4}
\end{equation*}
$$

In the case of acrylic acid/methyl acrylate copolymers, the elements of the set $\cap S_{1}$ are in the areas marked by the dashed line in Table 2. $r_{1}$ and $r_{2}$ values computed according to Relations (4) are

TABLE 2, $\mathbf{r}_{1}, \mathrm{r}_{2}$ Values Com-

| Copoly- |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 2 |  | 3 |  | 4 |  |
| $r_{1}$ | $\mathrm{r}_{2}$ | $\mathrm{r}_{1}$ | r 2 | $\mathrm{r}_{1}$ | $\mathrm{r}_{2}$ | $\mathrm{r}_{1}$ | r 2 |
| 1.076 | 0.173 | 0.894 | 0.051 | 0.809 | 0.075 | 0.361 | 0.043 |
| 1.153 | 0.806 | 0.938 | 0.106 | 0.847 | 0.246 | 0.507 | 0.251 |
| 1.241 | 1.175 | 0.985 | 0.256 | 0.887 | 0.341 | 0.701 | 0.426 |
| 1.340 | 1.490 | 1.036 | 0.354 | 0.930 | 0.407 | 0.822 | 0.555 |
| 1.453 | 1.733 | 1.091 | 0.459 | 0.976 | 0.441 | 0.926 | 0.703 |
| 1.584 | 2.493 | 1.150 | 0.652 | 1.025 | 0.513 | 1.008 | 0.768 |
| 1.737 | 2.830 | 1.214 | 0.779 | 1.078 | 0.588 | 1.143 | 0.951 |
| 1.917 | 3.617 | 1.283 | 0.965 | 1.135 | 0.707 | 1.302 | 1.034 |
| 2.135 | 4.419 | 1.359 | 1.066 | 1.196 | 0.836 | 1.423 | 1.266 |
| 2.401 | 5.404 | 1.443 | 1.286 | 1.263 | 0.976 | 1.490 | 1.318 |
| 2.735 | 5.404 | 1.534 | 1.742 | 1.335 | 1.127 | 1.717 | 1.486 |
| 3.165 | 7.228 | 1.636 | 1.892 | 1.415 | 1.293 | 1.996 | 1.744 |
| 3.741 | 10.387 | 1.748 | 2.053 | 1.501 | 1.674 | 2.222 | 2.141 |
| 5.780 | 14.198 | 1.874 | 2.317 | 1.597 | 1.973 | 2.488 | 2.650 |
|  |  | 2.016 | 2.226 | 1.702 | 2.227 | 2.641 | 2.770 |
|  |  | 2.177 | 3.325 | 1.819 | 2.318 | 2.994 | 3.492 |
|  |  | 2.361 | 3.918 | 1.950 | 2.614 | 3.985 | 4.771 |
|  |  | 2.574 | 4.262 | 2.098 | 3.069 | 4.711 | 6.070 |
|  |  | 2.822 | 4.850 | 2.265 | 3.766 | 5.704 | 7.448 |
|  |  | 3.116 | 5.549 | 2.457 | 4.096 | 7.143 | 11.478 |
|  |  | 3.469 | 6.391 | 2.678 | 4.662 | 11.129 | 26.667 |
|  |  | 3.902 | 6.712 | 2.937 | 4.872 |  |  |
|  |  | 4.444 | 7.056 | 3.242 | 5.855 |  |  |
|  |  | 5.144 | 10.427 | 4.060 | 8.393 |  |  |
|  |  | 6.080 | 14.749 | 4.624 | 8.393 |  |  |
|  |  | 9.394 | 23.349 | 5.352 | 11.222 |  |  |
|  |  |  |  | 9.774 | 25.184 |  |  |

puted by Means of MEMØRY 7/1
mer no.

| 5 |  | 6 |  | 7 |  | 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{r}_{1}$ | r 2 | $\mathrm{r}_{1}$ | $\mathrm{r}_{2}$ | $\mathrm{r}_{1}$ | $\mathrm{r}_{2}$ | $\mathrm{r}_{1}$ | $\mathrm{r}_{2}$ |
| 0.212 | 0.009 | 0.090 | 0.114 | 0.126 | 0.276 | 0.174 | 0.556 |
| 0.414 | 0.212 | 0.244 | 0.223 | 0.242 | 0.328 | 0.314 | 0.629 |
| 0.520 | 0.275 | 0.397 | 0.343 | 0.373 | 0.403 | 0.374 | 0.656 |
| 0.617 | 0.345 | 0.440 | 0.414 | 0.488 | 0.420 | 0.405 | 0.684 |
| 0.758 | 0.444 | 0.510 | 0.473 | 0.519 | 0.420 | 0.470 | 0.713 |
| 0.822 | 0.488 | 0.728 | 0.635 | 0.616 | 0.493 | 0.538 | 0.744 |
| 0.965 | 0.558 | 0.860 | 0.717 | 0.797 | 0.579 | 0.646 | 0.848 |
| 1.132 | 0.782 | 0.972 | 0.746 | 0.836 | 0.602 | 0.981 | 0.972 |
| 1.388 | 0.966 | 1.142 | 0.912 | 0.960 | 0.627 | 1.077 | 1.019 |
| 1.447 | 0.966 | 1.287 | 1.071 | 1.004 | 0.653 | 1.127 | 1.019 |
| 1.643 | 1.076 | 1.395 | 1.116 | 1.145 | 0.738 | 1.463 | 1.123 |
| 1.960 | 1.319 | 1.452 | 1.211 | 1.411 | 0.837 | 1.590 | 1.180 |
| 2.149 | 1.555 | 1.512 | 1.211 | 1.560 | 0.953 | 1.872 | 1.243 |
| 2.364 | 1.691 | 1.641 | 1.316 | 1.799 | 0.996 | 2.029 | 1.382 |
| 2.611 | 1.765 | 1.710 | 1.316 | 1.949 | 1.041 | 2.288, | 1.461 |
| 3.059 | 2.202 | 2.024 | 1.562 | 2.111 | 1.090 | 3.033 | 1.744 |
| 3.426 | 2.417 | 2.113 | 1.562 | 2.382 | 1.196 | 3.579 | 2.128 |
| 4.123 | 3.278 | 2.526 | 1.872 | 2.584 | 1.255 | 4.443 | 2.467 |
| 4.724 | 4.408 | 2.774 | 1.962 | 2.924 | 1.318 | 5.096 | 2.674 |
| 5.082 | 5.046 | 2.911 | 2.059 | 3.785 | 1.537 | 5.889 | 3.188 |
| 6.487 | 5.424 | 3.386 | 2.273 | 3.959 | 1.622 | 6.519 | 3.515 |
| 7.854 | 6.904 | 3.984 | 2.661 | 4.341 | 1.715 | 9.778 | 4.979 |
| 11.201 | 11.809 | 4.763 | 4.097 | 5.016 | 2.182 | 10.451 | 5.742 |
| 15.324 | 24.345 | 5.428 | 4.404 | 5.554 | 2.331 | 13.015 | 8.166 |
|  |  | 6.759 | 6.139 | 6.536 | 2.685 | 14.115 | 8.116 |
|  |  | 7.330 | 6.139 | 7.813 | 3.742 |  |  |
|  |  | 10.770 | 11.057 | 8.330 | 4.126 |  |  |
|  |  |  |  | 10.267 | 5.146 |  |  |

TABLE 3. Sequence Distribution ${ }^{\text {a }}$

| 1 |  |  |  |  | 2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | a |  | b |  | a |  | b |  | c |  |
| k | $\mathrm{n}_{1}(\mathrm{k})$ | $\mathrm{n}_{2}(\mathrm{k})$ | $\mathrm{n}_{1}(\mathrm{k})$ | $\mathrm{n}_{2}(\mathrm{k})$ | $\mathrm{n}_{1}(\mathrm{k})$ | $\mathrm{n}_{2}(\mathrm{k})$ | $\mathrm{n}_{1}(\mathrm{k})$ | $\mathrm{n}_{2}(\mathrm{k})$ | $\mathrm{n}_{1}(\mathrm{k})$ | $\mathrm{n}_{2}(\mathrm{k})$ |
| 1 | 24 | 135 | 21 | 110 | 264 | 174 | 126 | 81 | 14 | 7 |
| 2 | 18 | 6 | 13 | 20 | 30 | 74 | 52 | 48 | 10 | 6 |
| 3 | 19 |  | 14 | 1 | 3 | 32 | 21 | 29 | 3 | 1 |
| 4 | 12 |  | 12 |  | - | 12 | 7 | 18 | 5 | 5 |
| 5 | 10 |  | 12 |  | - | 4 | 1 | 6 | 3 | 4 |
| 6 | 14 |  | 15 |  | 1 | 1 | 1 | 9 | 5 | 2 |
| 7 | 8 |  | 5 |  |  | - | 1 | 6 | 4 | 2 |
| 8 | 2 |  | 6 |  |  | 1 |  | 4 | 4 | 2 |
| 9 | 3 |  | 2 |  |  |  |  | 2 | 2 | 1 |
| 10 | 8 |  | 6 |  |  |  |  | 3 | - | 4 |
| 11 | 4 |  | 4 |  |  |  |  |  | 2 | 1 |

${ }^{a_{\text {T }}}$ The reported values were obtained by means of MEMØRY-3 and they are slightly different when compared with those obtained by MEMめRY-7/1:

$$
\begin{array}{ll}
1-a: r_{1}=1.07, r_{2}=0.173 ; & 2-a: r_{1}=0.24, r_{2}=0.33 \\
1-b: r_{1}=1.24, r_{2}=1.17 ; & 2-b: r_{1}=1.41, r_{2}=0.84 \\
& 2-c: r_{1}=10.27, r_{2}=5.15
\end{array}
$$

$r_{1}=17.857 / 13=1.374$ and $r_{2}=13.492 / 13=1.038$, and compare very well with those determined by Eldridge and Treloar [9], $r_{1}=1.4$ and $r_{2}=1.0$ (these values are reliable, for they were calculated by the method of Fineman and Ross and checked by the methods of TidwellMortimer and Tüdös).

Because information concerning the sequence distribution is not available, we could not use Method 1 mentioned above. In order to argue the statement that macromolecules discriminate well in sequence distribution space, the corresponding data for two copolymer compositions are collected in Table 3.

To conclude, the present paper proposed and applied with satisfactory results an original method to compute the reactivity ratios in binary irreversible copolymerization. With some technical improvements the method may be of practical value. We note that the
method described satisfies the conditions pointed out by Behnken [10] and Tidwell and Mortimer [11], i.e., the method does not transform the error structure in the observed copolymer composition (for an elegant study of this problem, see Refs. 12 and 13).

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